High Frequency Motion Residuals in Multibeam Data: Identification and Estimation

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Abstract

Advances in multibeam sonar mapping and data visualization have increasingly brought to light the subtle integration errors remaining in bathymetric datasets. Traditional field calibration procedures, such as the patch test, just account for static orientation bias and sonar to position latency. This, however, ignores the generally more subtle integration problems that generate time-varying depth errors.

Such dynamic integration errors are the result of an unknown offset in one or more of orientation, space, sound speed or time between the sonar and ancillary sensors. Such errors are systematic, and thus should be predictable, based on their relationship between the input data and integrated output. A first attempt at addressing this problem utilized correlations between motion and ping-averaged residuals [1]. The known limitations of that approach, however, included only being able to estimate the dominant integration error, imperfectly accounting for irregular sounding distribution and it only working in shallow water.

This paper presents a new and improved means of utilizing the dynamics of the integration error signatures which can address multiple issues simultaneously, better account for along-track sounding distribution, and is not restricted to the shallow water geometry. The motion-driven signatures of multiple integration errors may be simultaneously identified through individually considering each sounding’s input-error relationship along extended sections of a single swath corridor. Such an approach provides a means of underway system optimization using nothing more than the bathymetry of typical seafloors acquired during transit. Successful estimation, however, imposes conditions of significant vessel motion, and smooth, gently rolling bathymetry. Initial results of the new algorithm are presented using data generated from a simulator, with known inputs and integration errors, to test the efficacy of the method.

I – Introduction

Multibeam bathymetric mapping, a marine acoustic remote sensing technique, necessarily requires the integration of platform orientation and position measurements with array-relative ranges and angles in order to georeference remote seafloor interactions. While individual sensors making such measurements may be calibrated to superb precision and accuracy, their offsets in space, orientation and time relative to the sonar, are often difficult to measure when installed separately on that platform [2]. Those integration parameters beyond the core set estimated by the patch test are typically ignored, as their influence on the final solution is usually just within
the allowable total accuracy limits. While small, however, these errors nonetheless often are present in bathymetry [1], propagating as high-frequency, motion dependent depth errors (Fig. 1).

Figure 1: Sun illuminated bathymetry of wobble observed in field data acquired in approximately 130m of water, resulting from an unknown integration error. The artificial signature, oriented transverse to the ship track, is notably of similar scale and wavelength to that of the fine detail bed-forms also present in the image, though approximately parallel to ship track here. Such artificial signatures can significantly obstruct the analysis of fine-scale relief, particularly if unfortunate enough to be parallel to the superimposed artefact. Data courtesy of NOAA Ship Thomas Jefferson.

This paper first reviews six integration errors common to bathymetry [1] and the nature of their propagation from the raw auxiliary sensor input in shallow and deep water conditions. It follows with a summary of the relevant literature regarding the calibration of swath bathymetric sonar systems, and more generally swath systems, of which methods developed for lidar are notably applicable. A new approach which simultaneously estimates the six integration errors in the presence of gently rolling bathymetry at all depths is then presented. This is referred to herein as the rigorous inter-sensor calibrator (RISC). Finally, simulated results acquired over a seafloor modelled at various depths and wavelengths are analyzed when driven by both synthetic and real vessel motion to assess the method’s capability.
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1.1 – Errors Remaining in Integration

High-frequency, motion-induced errors resulting from latency and scaling of INS output were first analyzed in the early nineties by [3]. A deeper investigation into such motion dependent errors was undertaken by [1], where the impact of those integration errors, along with four additional common sources of significant, high-frequency depth errors were characterized:

1) GNSS-MBES X-lever error, $\Delta L(x)$
2) GNSS-MBES Y-lever error, $\Delta L(y)$
3) INS-MBES latency, $\Delta t$
4) INS scaling, $\Delta \rho$
5) INS-MB Z-axis misalignment, $\Delta \kappa$
6) SSS Error (latency and/or bias), $\Delta_{SSS}$

What level of error is significant here is defined as that greater than the particular sonar suite’s bottom-tracking noise floor, currently at best around 0.1-0.2% ($1\sigma$) of water depth [4]. This can vary widely according to depth-varying sonar configuration settings, including pulse length and type, as well as incidence angle, which all impact the bottom detection algorithm [4].

Each of the integration errors presented here acts to offset the estimated origin, and in some cases the orientation of the multibeam, or more specifically it’s steered, virtual acoustic origin and beam vector. This is caused by erroneously applying the auxiliary sensor data to the multibeam in integration. These errors were shown by [1], to each be primarily driven by unique components of platform motion (Fig. 2). As a result, theoretically they should be uniquely identifiable in soundings acquired by a vessel undergoing dynamic angular motion.

![Figure 2: The impact of each of six integration errors presented in Hughes Clarke, (2003), at their symmetric error extrema when oscillating with a ten degree amplitude. Each subfigure number corresponds to each integration error in the above list.](image-url)
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The various errors in the modelled beam start point and vector then propagate to the integrated sounding as a three-dimensional position error (Fig. 3). Those integration errors propagating as an imperfectly orientated multibeam, namely errors 3 to 6 in the above list and table, scale with range and increase with obliquity. In contrast, those causing only beam vector start point errors, errors 1 and 2, are not depth scaling and thus decrease in importance with depth.

Any resultant georeferenced sounding is derived from paired geometric components at transmit and receive time (Fig. 3). At those times, an INS/IMU provides orientation triplets and a GNSS antenna provides position triplets, which are used to orient the array-relative steering angle, with respect to the multibeam’s transmitter and receiver at their respective epochs.

As all the considered errors scale with the instantaneous orientation, the net result for periodic motion is a correspondingly periodic imprint of that error on the seafloor (Fig. 3) with a projected length scale which reflects the vessel displacement over the wave period. For example, a projected wavelength of 40 meters would result from a five m/s vessel speed and eight second wave period. In shallow water, where the shot-receive cycle is far shorter than the wave period, the projected wavelength undulates almost entirely along-track.

![Figure 3: Illustration of geometry required for integrating a sounding: The multibeam’s array-relative angles, orientation triplets as observed by an INS/IMU, and three dimensional positions observed by a GNSS antenna. These are each required at every sounding’s transmission and reception, with two separate arrays observing the transmit and receive array-relative angles. Transmit and receive epoch separation, as well as the vertical axis scale are hugely exaggerated here for illustration. The resultant oscillating surface overprint resulting from an erroneous integration are shown as well as the specific instantaneous depth and horizontal position error. As depth increases, the change in the sonar’s position and orientation over the shot-receive cycle increases as a result of pings seeing increasing travel time from source to seabed and back to receiver. This causes the character of the embedded integration errors to change, complicating the analysis of the depth residuals. In “shallow” water (50m in Fig. 4a), the manifest error is effectively constant for the entire ping, and thus the wobble appears orthogonal to the ship’s track. In “deep” water (5000m in Fig. 4b), however, the error is clearly seen to evolve over the ping cycle, and thus the errors are no longer exactly orthogonal to the ship track, migrating obliquely. As a result, the wobble has a projected undulation both along, and now, across-track.](image-url)
Figure 4: Bathymetric signature of each of the six integration errors when uniquely driven by eight second, three-degree sinusoids sequentially in roll, pitch and yaw as well and finally a one meter heave over a planar seafloor at a) 50m; b) 5000m. Integration error magnitudes are such that they propagate as depth errors with peak amplitudes of approximately \( \frac{1}{4} \) a percent of water depth (% W.D.). Along-track distance in b) is increased approximately an order of magnitude to illustrate deep water trends.

It is clear that, as the water depth becomes greater the manifested depth error becomes increasingly nonlinear across-track, with wobble evolving significantly as the vessel dynamically oscillates throughout the shot-receive cycle. This is the reason that the method of [1] was explicitly restricted to shallow water, where the ping period is short relative to the wave period. The method described herein, however, uniquely considers the relationship between each sounding’s error and the vessel state both at transmit and reception, removing this restriction.

1.2 – Existing Swath Calibration Methods

In addition to characterizing the impact of each integration error, the first attempt to quantify these errors [1] presented a computationally efficient calibration procedure, estimating the errors directly from a single swath corridor. The assumption was that the corridor contained no real bathymetric roughness with dimensions close to the likely embedded integration wobble. The approach opportunistically analyzed suitable swath corridor extents, by testing for and rejecting areas containing such “roughness”, thereby producing increased redundancy in integration error estimates. This approach has the notable advantage of not requiring additional survey lines for calibration. The errors were estimated by assuming their signatures to be either across-track tilts or vertical departures of each ping from the running average, and linearly correlating that with each error’s main driver existing at transmission, either: roll, roll rate, pitch or heave.

To be effective, the approach in [1] required:

A. the regional seafloor depth and slope be suitably removed from the analysis,
B. the sonar angular or depth anomaly being approximately constant for each beam across the swath,
C. only one of the highly correlated error signatures existing (or at least dominating).

Requirement A is general for regression, where some suitable reference of truth is sought. This was addressed in [1] by using across-track regressions of each ping, producing a series of depths and slopes, which were then high-passed filtered to remove the long wavelength seafloor signature, assumed to be real. This assumed the seafloor to be locally planar, and the seafloor’s sampled across-track slopes and depths to only be changing over time constants long with respect to the wave period. The locally planar condition is often reasonable for unconsolidated
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sedimented seafloors, but limits the class of seafloors suitable for calibration. The long time period assumption is not strictly true for a vessel yawing or pitching on an incline \([1]\), and even more so with multi-sector stabilization (Fig. 5). Requirement B holds in shallow water conditions, such as 50 meters, however, begins failing before 500m as a result of the shot-receive cycle becoming significant relative to the driving wave period \([1]\). Finally, requirement C is an ideal situation, next to of course there being no errors at all. Failure of either of these significantly degrades results. The method developed here seeks to overcome these limitations, producing a calibration method more robust to the various classes of bathymetry acquired by ocean mappers.

Figure 5: Output of multi-sector simulator developed to test the RISC approach. Across-track discontinuity of soundings occurring at sector boundaries on gently rolling relief. This is particularly relevant to the triple sector systems employed by much of NOAA’s current OCS fleet.

Most literature on sonar integration calibration is geared towards identifying the sources of the, often more significant, static biases in observations. This has guided the development of field calibration routines which implement well-designed overlapping swath corridors to identify underlying errors \([5], [6], [7], [8], [9], [10], [11], [12])\). The general approach for adjustment is to minimize the deviation of soundings, either by eyeing point clouds \([5], [6], [7], [8], [10]\), regressing onto a planar surface \([9]\), and more recently, onto a quadratic surface \([11], [12], [13]\).

Subsets of the dynamic integration errors investigated herein, particularly lever arms and INS latency, have been analyzed in sonar \([9], [11], [12], [13]\), as well as lidar \([14], [15], [16], [17], [18]\). With the exception of \([13]\), in the case of sonar, only the patch test parameters of IMU-
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MBES boresight misalignment and GNSS-MBES latency are considered. For that subset, even though the errors do have inter-dependencies, which might require simultaneous estimation to mitigate inter-error leakage, this is not necessarily required for the patch test. Through the use of carefully paired line geometries and specific stepwise estimation and application of the calibration parameters, errors other than that of current interest cancel out ([4], [5], [7], [8], [10]). In contrast to the simpler patch test approaches, the referenced lidar methods, and more recent sonar approaches ([9], [11], [12], [13]), implement a mathematical model coupling the error terms. A gradient method, such as iterative least squares, is then applied for nonlinear optimization relative to some reference of truth. The sonar approaches, apart from [13], all implement overlapping swaths to define said reference, however, which the method described here seeks to avoid.

Another finding first noted by the lidar community, [15] demonstrates that the smooth extensive surfaces often found in nature can be effectively modelled by fitting low order polynomials over extended spatial extents of scattered measurements acquired by lidar. Such surfaces are thus often suitable for calibration. The approach, however, required *apriori* definition of surfaces using higher accuracy RTK GNSS observations, which pioneering ocean mapping missions do not have the luxury of. Extension to sonar systems has seen the use of in-situ data to simultaneously estimate such surface models ([11], [12], [13]).

The most recent multibeam calibration procedure presented in [13] implements an approach which, similarly to that in [1], estimates the integration error using only a single swath corridors’ soundings. Whereas [1] implements two-dimensional seafloor detrending by high pass filtering a series of across-track fitted lines, [13] fits a quadratic surface along the swath corridor. A mathematical integration model is necessarily employed to account for the evolution of the propagated error signature over the extended along-track domain. This uniquely analyzes each sounding’s input-error relationship within the domain. An iterative least squares adjustment is then implemented to optimize the sounding depth misclosures as a function of latency. Notably, however, this is the only parameter solved by the method. In contrast the approach presented here, addresses six parameters simultaneously.

**II – Rigorous Inter-Sensor Calibrator (RISC)**

Considering the high frequency signatures of the propagated motion-dependent integration errors, [1] and [13] proposed that a single swath corridor is often sufficient for analysis, once underlying assumptions of suitable seafloor and vessel motion are achieved. Wave-driven, high-frequency vessel motion and low frequency seafloor undulation combine to produce conditions in which each integration error’s propagated “wobbles” quickly decouple from each other, as well as the seafloor. A requirement for calibration through regression, as is presented here, is to have a suitable reference of the true seafloor. An algorithm is designed here which simultaneously identifies the six integration errors, while accurately modelling suitable seafloors.

**2.1 – Coupling of Errors with a Georeference Model**

Though [1] demonstrated the correlated errors can often be identified and solved in a stepwise manner, starting with the most significant, this approach seeks to simultaneously estimate each. The across-track depth error becomes a nonlinear function of each of the integration errors when
multiple errors are present and thus a coupled model is required to properly distinguish many of them.

The coupling of integration errors for calibration of scattered swath systems was first introduced in the lidar community ([14], [15], [16], [17], [18]) by implementing various stochastic models to minimize the spread of those scattered measures relative to, often planar, surface features. A georeference model coupling the integration errors provides a means of analyzing the relationship between the input to each point measurement and the resultant manifest error. By utilizing multiple points, typically several thousand, over an extended domain such as a swath corridor, this relationship typically increases over-determination of calibration parameters. Such an approach is equally suited to both shallow and deep water conditions, where the realization of the depth error begins to evolve across individual pings. Furthermore, this approach is able to manage the dependent nature of the integration errors discussed here, as regression using the coupled model can consider the correlations among them. Published sonar calibration techniques ([9], [11], [12]) have implemented a similar approach, which is followed here. The geometric form of any remote observation is quite familiar:

\[
X = MB_{MRF} + s \begin{bmatrix} \cos(\phi + \phi_R) \cos \theta \\ \cos(\phi + \phi_R) \sin \theta \\ \sin(\phi + \phi_R) \end{bmatrix}
\]

where:
- \(X\) = Sounding coordinate,
- \(MB_{MRF}\) = Position of multibeam’s virtual acoustic center in mapping reference frame,
- \(s\) = Slant range to sounding
- \(\phi\) = Initial depression angle of transmission,
- \(\phi_R\) = Additional depression angle to sounding due to water column refraction,
- \(\theta\) = Azimuth of transmission.

Every sonar integration package has to have some form of the above georeferencing implementation, although it is not typically accessible from proprietary packages. To be specifically useful for least squares optimization, however, the implementation has to allow for the partial derivatives of the integrated sounding relative to each integration error to be accessible. This requires the equation to be analytical.

This paper will use a swath corridor simulator to produce ideal datasets for analysis. This simulator implements a concentric intersection of the transmitter and receiver’s cones of sensitivity which accounts for the non-orthogonality between them, such as that seen in [19], illustrated in Figure 6. This generic georeference model implies the signal follows the same path to and from the seafloor, requiring only one sequence of refractions for integration. This necessarily discrete ray trace through the assumed horizontally stratified water column is accounted for using \(s\) and \(\phi_R\). Implementation of these two variables, calculated during the ray-trace procedure, produces an analytical form of the georeference model, making it suitable for optimization.
This simplified concentric model is sufficient for analyzing the characteristics of the integration error signatures, particularly in a simulated environment as presented here, where the true sounding positions are actively defined by the implemented model. Simulation provides a closed environment ideal for analyzing the “wobble” imposed on bathymetry when acquired under various input “designs”, here meant to represent environmental conditions. Results can then be used to assess the proposed method’s theoretical capability to simultaneously determine the sources of wobble in various classes of environment, in particular water depth, seafloor undulation wavelength, and angular vessel motion magnitudes and rates. This is possible through comparison to their known, forced values.

Sensor input includes:

- positions in a mapping reference frame (MRF), here given by GNSS,
- triplets of Tait-Bryan angles defining orientation, \((\omega, \phi, \kappa)\),
- array relative angles dictating the directions sound is transmitted and received,
- sound speed profile (SSP) and two-way travel time (TWTT) measures.

Each of these classes of measure, apart from TWTT, must be determined at both the epochs of signal transmission and reception for accurate integration. The TWTT is implicit to the active remote sensing procedure, and as a result, is the most difficult to calculate in simulation. Every other input can be trivially parameterized for simulation, with the seafloor, component angular motions and heave all being defined here as sinusoids.
The underlying seafloor relief sinusoid is simply two-dimensional and spatially parametrized to allow for undulation in any direction, while motion is temporally parameterized. As horizontal vessel motion is not of interest to this study, it is simply temporally parametrized as a line. The SSP is parametrized as a function of depth, having constant gradient and is depth-interpolated for perfect SSS values at the multibeam array face. Array relative angles are calculated such that sectors are steered to ideal geographic angles as a function of the sonar pitch and heading at transmission and roll at reception. Finally, TWTT is then brute forced in simulation by tracking the resulting refracted ray path until it intersects the model seafloor.

The integration errors’ impact on bathymetry can be characterized by their effect on the position and orientation observations transformed from the auxiliary GNSS antenna and INS/IMU respectively, to the multibeam’s reference frame. This reference frame is a virtual combination of the transmitter and receiver at their respective measurement epochs. The parametrically-linear impact of each integration error was already presented in [1]. Those same errors are here coupled into the sounding georeference model, through designing them into the auxiliary sensor inputs, which are transformed to the multibeam arrays for integration. For the sake of simplicity, assuming the transmitter, \( \mathbf{n}_{Tx} \), and receiver, \( \mathbf{n}_{Rx} \), to be respectively parallel and perpendicular to the vessel reference frame (VRF) eliminates the need for including their alignment relative to INS and gyro for integration, though this can be easily added.

\[
\begin{align*}
\omega^*_i &= \sin^{-1}\left(\cos \Delta \kappa \sin \left(\Delta \rho \left(\omega_i - \frac{\Delta \omega_i}{\Delta t_i} \Delta t\right)\right) + \sin \Delta \kappa \sin \left(\Delta \rho \left(\psi_i - \frac{\Delta \psi_i}{\Delta t_i} \Delta t\right)\right)\right), \\
\psi^*_i &= \sin^{-1}\left(\cos \Delta \kappa \sin \left(\Delta \rho \left(\psi_i - \frac{\Delta \psi_i}{\Delta t_i} \Delta t\right)\right) - \sin \Delta \kappa \sin \left(\Delta \rho \left(\omega_i - \frac{\Delta \omega_i}{\Delta t_i} \Delta t\right)\right)\right), \\
\kappa^*_i &= \kappa_i - \frac{\Delta \kappa_i}{\Delta t_i} \Delta t, \\
Hv^*_i &= \Delta \rho \left(Hv_i - \frac{\Delta Hv_i}{\Delta t_i} \Delta t\right),
\end{align*}
\]

where:

- \( (\omega^*_i, \psi^*_i, \kappa^*_i) \) = Adjusted orientation triplet measured by auxiliary INS/IMU sensors,
- \( Hv^*_i \) = Adjusted heave output by INS,
- \( \left[\frac{\Delta \omega_i}{\Delta t_i}, \frac{\Delta \psi_i}{\Delta t_i}, \frac{\Delta \kappa_i}{\Delta t_i}, \frac{\Delta Hv_i}{\Delta t_i}\right] \) = roll, pitch, heading and heave rates via raw high frequency input.

It is then the above erred output angles which are applied in both orienting the multibeam and positioning it relative to the position source, typically either the GNSS antenna or vessel reference point (RP). This gives the following equation for position of the virtual, concentric multibeam array, as an average of the transmitter and receiver positions at their respective epochs:

\[
MB^MRF^*_{i, MRF} = \frac{1}{2} \left(\mathbf{GNSS}^MRF_{i, Tx} + \Re(\kappa^*_i, \psi^*_i, \omega^*_i)_{Tx}\left(-\mathbf{GNSS}^VRF_{i, Tx} + Tx^VRF - \Delta L\right)\right) + \frac{1}{2} \left(\mathbf{GNSS}^MRF_{i, Rx} + \Re(\kappa^*_i, \psi^*_i, \omega^*_i)_{Rx}\left(-\mathbf{GNSS}^VRF_{i, Rx} + Rx^VRF - \Delta L\right)\right),
\]

where:

- \( \mathbf{GNSS}^MRF_{i, Tx/Rx} \) = position in mapping reference frame (MRF) at transmission/reception,
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- $\mathbb{R}(\kappa_T^*, \psi_T^*, \omega_T^*)_{TX/RX} =$ rotation from transmitter’s/receiver’s reference frame to MRF using adjusted orientation triplet (heading, pitch, roll),
- $-\text{GNSS}^{VRF} + \text{T}x^{VRF}/\text{Rx}^{VRF} =$ GNSS-transmitter/receiver lever arms in VRF.

Finally, the actual steered array-relative angles, that are combined to geographically orient the transmitted signal, are themselves impacted by surface sound speed estimates in the following manner:

$$\begin{align*}
\theta_T^* &\approx \sin^{-1} \left( \frac{\text{SSS} - \Delta_{\text{sss}}}{\text{SSS}} \sin \theta_T \right), \\
\theta_R^* &\approx \sin^{-1} \left( \frac{\text{SSS} - \Delta_{\text{sss}}}{\text{SSS}} \sin \theta_R \right).
\end{align*}$$

A consequence, however, of using a flat array in a horizontally stratified fluid is that the errors in the estimated array-relative steering angles are almost entirely corrected when the sound speed used in refraction converges to the truth, typically at depth ([1], [20]). Using a snapback layer refraction at the array face, as described in [20], and conveniently assuming a perfect sound speed profile, this correction, [21], adjusts the error in the steered portion of the geographic depression angle. The result is the following terms are used to estimate the propagation of a purely SSS error:

$$\begin{align*}
\theta_T^* &\approx \sin^{-1} \left( \sin \left( \theta_T - \psi_T \left( 1 - \sqrt{\sin^2 \theta_R} \right) \right) + \frac{\text{SSS}_{\text{Est}} - \Delta_{\text{sss}}}{\text{SSS}_{\text{Est}}} \sin \psi_T \left( 1 - \sqrt{\sin^2 \theta_R} \right) \right), \\
\theta_R^* &\approx \sin^{-1} \left( \sin \left( \theta_R - \omega_R \right) + \frac{\text{SSS}_{\text{Est}} - \Delta_{\text{sss}}}{\text{SSS}_{\text{Est}}} \sin \omega_R \right).
\end{align*}$$

These terms can be integrated into any georeference equation and the soundings’ sensitivities with respect to the parameters can be analyzed for optimization. Notably the along-track error component is a simplification of the following along-track motion compensation angle, implemented in simulation, when zero yaw is assumed:

$$\theta_T = \arctan \left( -\sin \kappa_T \sin \theta_T + \cos \kappa_T \sin \psi_T \left( 1 - \sqrt{\sin^2 \theta_T} \right) \right).$$

This angle is derived as that required to shift a cone aligned on the transmitter, $\mathbf{n}_{TX}$, from a given along-track angle to nadir. This along-track angle is relative to the course made good, here taken to be North, or 0°.

2.2 – Suitable Truth; Flattening Residuals to Local Natural Surfaces

In order to have a good estimate of each instantaneous beam depth error, a model of the true seafloor is required as a reference. In the absence of an independent truth, such as an overlapping perfect survey, this approach attempts to extract the true seafloor from the imperfect observations themselves. The key is to have a seafloor model that is immune to the sought imperfections in the observations, and of course a seafloor which the model suitably approximates. As discussed, any integration error will be projected with a characteristic spatial length scale, directed primarily along-track, and thus seafloors that contains only depth variations over significantly longer wavelengths are sought.

2.2.1 – The Seafloor as a Quadratic

While [1] reasonably removed the seafloor using just an across-track linear regression and an along-track high pass filter, it has been demonstrated by [13] that a quadratic surface may be
used to estimate the underlying natural seafloor surface using a swath corridor containing high
frequency bathymetric errors. This surface simultaneously filters along-track and across-track
seafloor trends. Using a surface as opposed to a line feature further allows for integration error
estimates to be made using extents of swath corridor as opposed to a single ping, thereby
increasing domain size. This generally produces more stable solutions while simultaneously
better accounting for the along-track irregularity of the seafloor, sampled from a dynamically
oscillating vessel. This requires an error recovery model which analyzes the relationship between
each sounding’s input and error, as the manifest integration error is no longer constant within the
domain.

This paper proposes a combination of a quadratic model as the estimate of the true surface,
together with the presented coupled georeference model, so that error estimation can proceed.
This produces an equation for regression which can account for the variation in the error’s
driving signature, predominantly vessel orientation, within the regression domain. Considering
the standard stochastic model:

$$\varepsilon = y - h(x, \beta).$$

Expanding the model and its disturbance terms to consider both the sounding and seafloor
components, and setting every observation to be necessarily zero, the following expression is
acquired for the error to be minimized, which thereby seeks to “flatten” the soundings to the
quadratic surface:

$$(\varepsilon_s - \varepsilon_q) = 0 - \left( f(x_s, \beta_s) - g(x_q, \beta_q) \right) = \Delta z,$$

where:

- $f(x_s, \beta_s) = MB^{MRF^*}(z) + s \sin(\phi + \phi_R)$.
- $g(x_q, \beta_q) = \beta_0 + \beta_1x + \beta_2y + \beta_3xy + \beta_4x^2 + \beta_5y^2$.

Uniquely considering each sounding’s misclosure (Fig. 3) has the added benefit of eliminating
the need to limit analysis to shallow water conditions, as is the case in [1], as the temporal
evolution of the propagated “wobble” among the soundings is now accounted for. As a result, the
temporal extent of soundings analyzed locally can be significantly extended to encompass the
long shot-receive cycles at depth.

The desire for long wavelength undulation is twofold. First, it is characteristically unique from
short wavelength wobble, and second, it is expected to have slowly varying slope and curvature,
the very parameters the quadratic surface seeks to estimate. Considering the quadratic surface as
a polynomial filter, and sinusoidal seafloor undulation, the shortest wavelength seafloor
undulation which is expected to be suitable for analysis is twice as long as the window length,
across-track and along-track. This is in accordance with Nyquist folding frequency, which sees
low-order polynomials removing half the spectral power (3dB) of any undulation with
wavelength twice as long as the local moving window [22], here, a two dimensional quadratic,
and removing increasingly more power for longer wavelengths. The subtlety of the depth errors
typically expected calls for high accuracy seafloor models, as they can be easily obscured by
remnant seafloor undulation and roughness. Long-wavelength undulations, preferably many
times more than twice the window length are thereby recommended.

Figure 7 presents a quadratic surface fit to a sinusoid gently rolling along-track with a two
kilometer wavelength and 25 meter amplitude. The proposal is that a window length of 1 km
should remove half the power, or intensity, of that undulation, with the other half leaking into the

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analyzed depth residuals. For the subtle errors being analyzed, however, a far shorter window is desired to better fit, and thus further filter the seafloor undulation. Here, a window length of three wave periods, or 120 meters when steaming at 5 m/s is implemented, and found to suitably adhere to the underlying sinusoid. This is assessed by the convergence of the depth misclosures relative to the quadratic surface, to the true depth misclosures relative to the underlying sinusoid, available in simulation. Their convergence is illustrated top right. Notably, too short of a window will begin to follow the wobble itself, producing inaccurate estimates of the true depth misclosures.

![Figure 7: Swath corridor simulated with 6 simultaneous integration errors and driven by the motion time series illustrated top left. A subsection of wobbled swath corridor, spanning three wave periods, or 120 meters (illustrated as relief-colored crosses) is used to approximate the true underlying seafloor (illustrated as a mesh). Top right illustrates the ratio of depth misclosures estimated relative to the quadratic surface versus the sinusoid, taken as truth. A zero magnitude surface illustrates a strong quadratic fit.](image)

It is important to realize here that a quadratic surface, with only one inflexion and associated curvature across-track and another along-track, can never perfectly fit a sinusoid, as there are multiple inflexions and the curvature constantly changes. Thus from a quadratic fit point of view, the domain would ideally be no more than about ¼ of the sinusoid wavelength. A notable advantage implicit to using a quadratic surface here is that it is restricted to one inflexion along-track and another across-track, thereby quickly decoupling from the oscillatory motion-dependent errors projected onto the seafloor.

2.2.2 – A Need for Local Analysis

By considering extended domains, and thereby many thousands of point observations, typically the level of over-determination grows, and thus the confidence in the calibration parameters
estimates. Figure 8 illustrates estimate accuracies for a domain spanning two wave periods, here 16 seconds, versus four wave periods, or 32 seconds. Here all errors are simultaneously forced while the vessel heaves and oscillates in each component direction. This reassuringly illustrates that solutions appear to converge as more information is added to the system.

Figure 8: Here the rigorous inter-sensor calibrator (RISC) is applied to the wobbled soundings simulated over a planar seafloor at 500m depth. Vessel is oscillating in all three components as sinusoids with three degree amplitude and eight second period. All six errors are present with magnitudes indicated by the red line. The 16 and 32 second estimates are in purple and orange respectively, as well as their associated soundings. Boxplot axes are ±10% error magnitude in each case.

Typical seafloors, however, are not always planar. While a seafloor containing only long wavelength undulation is desired, seabed geology changes spatially, and the regional seafloor may contain undulations over a wide spectrum. This includes undulations with slowly evolving wavelengths, as well as regions where distinct features may suddenly arise in otherwise gently rolling seafloor, such as bedrock outcrops. The first case of a slowly evolving undulation wavelength should not pose much issue, already considering the wavelength is suitably long such that the seafloor slope and curvature do not significantly change over the regressed section of swath corridor. Fast changes in seafloor complexity, however, induced by the sudden appearance of such high frequency features, including sand waves, are likely unsuitable for analysis. This is because a simple quadratic model cannot account for variation in its trends over its domain. Unconsolidated sedimented surfaces, however, will generally meet this long wavelength requirement.

A consequence of using an approach which estimates the true seafloor with only non-overlapping swath corridors acquired in-situ, is that integration errors propagating as low
frequency, or static, depth errors are absorbed by the quadratic fit, and cannot be analyzed without suitable overlap of the swath corridor. Notably, remnant seafloor undulation not accounted for by the quadratic fit can cause artificial correlations of the integration errors with the depth misclosures. Natural roughness occurring at the scale of the projected wobble, expectedly more so.

Figure 9 illustrates a typical region of continental shelf bathymetry that exhibits a wide range of natural spatial wavelengths, some suitable, some not for this analysis. One can see both periodic integration artifacts (Fig. 9 short $\lambda$ A) as well as real natural roughness (Fig. 9 short $\lambda$ B) at similar length scales. There exists extensive regions, however, that do not contain that natural roughness. Examples shown are areas that are near flat (Fig. 9 planar), and those with medium or long wavelengths. These all arise within a three kilometer square region of seafloor. Thus, instead of optimizing an entire swath corridor, local regression of short sections is proposed. The simulated bathymetry used in this paper, reproduces sinusoidal morphology at these typical, longer length scales (300-700m).

![Image](image.png)

*Figure 9: Region of seafloor seeing rapid evolution in seafloor complexity as well as independent, and often overlapping undulations over a spectrum of wavelengths.*

2.2.3 – Local and Asymptotic Implementation of the RISC

Finally, the RISC approach operates by “flattening” the wobbled soundings to the smooth quadratic surface by seeking parameter sets that minimize the mismatch. The coupling of the integration errors naturally indicate that any optimization scheme applied to the measures, here depth misclosures, must be nonlinear. This is of course not the case if only one of the correlated errors is present, or at least dominant [1]. A simple unweighted, iterative least squares adjustment is carried out here in this proof of concept, though more sophisticated methods are typical in field implementation ([9], [11], [15], [16], [17], [18]).
A generalized smoothing approach is implemented similarly to along-track locally weighted scatterplot smoothing (LOWESS), where low-order polynomials, such as the quadratic surface considered here, are common ([23], [24]). To effectively exclude the wobble contribution from biasing the quadratic estimate of the true underlying seafloor undulations, using the same reasoning for defining the minimum suitable seafloor undulation wavelength as twice the window length, the along-track domain extent must be a minimum of twice the typical projected wobble wavelength. A window preferably four times as long is desired to retain suitably more than half the power of the wobble’s bathymetric signature [1], though this simultaneously increases the minimum seafloor undulation wavelength suitable for analysis.

For example, considering a typical projected wobble wavelength of 40 meters, a 160 meter window is expected to suitably retain the wobble, simultaneously defining the suitable minimum along-track undulation wavelength to be 640 meters. While such an extent is expected to contain tens of thousands of soundings in shallow water conditions, where the seafloor, and thus the propagated integration errors, are heavily sampled, because sounding density decreases as water depth increases, a longer window may be necessary at greater depths.

The across-track window length is here naturally limited to swath width. Notably, the system is similarly sensitive to across-track undulation, or more specifically, remnant seafloor trend, which correlate with the integration errors that propagate as across-track tilts (errors 3 to 6). Consequently, across-track undulations far greater than twice the across-track window length, or swath width, are strongly recommended. Assuming a flat seafloor, a swath width is approximately four times the water depth with a ±65° swath, or two kilometers in 500 meters of water. Notably, at depth, the wobble begins to evolve across-track, as a result of extended shot-receive periods. This may see increased correlation with high frequency across-track undulations.

While an integration error estimate can be achieved within a single domain, repeating this over successively offset sections of swath corridor adds information to the system, typically increasing accuracy and confidence levels. This is achieved simply through removing the most lagged ping in the current domain and appending the ping which leads it, after each local iterative least squares adjustment. This approach provides a series of smoothed integration error estimates, which are added to a growing, increasingly asymptotic average. This acts as regional smoothing, having a length scale which grows with time, and thus effectively becomes an asymptotic approach.

This smoothed “monitoring” of each integration error is particularly useful for parameters which may slowly drift spatiotemporally, such as surface sound speed bias. The asymptotic smoothing period for such nonstationary integration errors should simply be short relative to their drift, but long enough to enable suitable convergence to the true values.

In general, a significant number of soundings distributed over multiple phases of motion are desired. This is due to the similarities among the integration error’s input-depth misclosure relationships for a single orientation instance. This causes ambiguity in determining the signal’s integration error source, seeing reduced statistical significance of datasets. Soundings acquired when there is no vessel motion are of course no use to analysis, and Section 3.2 demonstrates how various combinations of vessel orientation, in addition to seafloor misfit, causes local estimates to “walk” about the true value. While a sufficient number of suitable soundings may
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not be acquired in a single domain, the asymptotic regional domain may attain this, assuming parameter values do not change before a stable convergence can be reached.

In a limiting sense, with little or no vessel motion, the sources of these dynamic residuals, cannot be distinguished from themselves, nor the underlying seafloor trends. Notably, sounding independence increases significantly with depth, and typically so does estimate accuracy for a given number of soundings. The total number of independent soundings over a given spatial extent, however, will not exceed the highly-sampled seafloor in shallow waters, and the expectance is that neither do estimate accuracies. With thousands of soundings collected every minute, approaching the order of tens of thousands in shallow water, there is no lack of data for analysis. Whether it is suitable for the method presented here, however, is addressed in the discussion.

III – Results and Discussion

This section seeks to identify the method’s limitations as a function of first seafloor depth and horizontal undulation wavelength, and then as combinations of component vessel motion frequencies. Swath corridors simulated using real angular motion and heave input are implemented to establish suitability of local and regional domain extents for a number of seafloor depths and undulation wavelengths. Herein, local refers to the along track length of the instantaneous quadratic fit, whereas regional refers to the distance over which the asymptotic average is accumulated. A detailed analysis of vessel motion follows, using the temporally parametrized component orientations previously discussed, and the seafloor undulation wavelengths which will be identified as suitable in Section 3.1, relative to a comfortable window length of four wave periods as recommended by [1].

The advantage in the method not requiring specific pairings of overlapping swaths is that enormous amount multibeam data typically acquired during mapping missions, and during their transits, may be implemented for analysis. There are expected regions of seafloor where a suitable reference surface cannot be established, and thus should not be considered for calibration. This paper does not seek to identify particular soundings most useful for analysis, rather to identify suitable characteristic seafloors and vessel motion. As a result, all soundings are considered for final estimates. The analysis implements a swath corridor simulated over a sinusoid oscillating along-track for approximately eight minutes, or a few kilometers, acquiring on the order of one million soundings in shallow water, varying of course with depth. Results then approach their asymptotic values, enabling the establishment on the lower limits of suitable seafloor undulation wavelengths.

Achieved parameter estimate accuracies then represent the theoretical capability of the model, considering perfect, designed input. Estimate uncertainties are not addressed here, which require the consideration of input uncertainties, dependence among the parameters and dependence of the input measures through time, their auto-correlation. Each case presented implements six simultaneously forced errors to assess the method’s capability when all are superimposed.

3.1 – Suitable Domain Extent

With the periodicity of open-ocean wind-driven gravity waves being in the spectrum of one and twenty five seconds ([25, 26]), having most of their power between four and twelve seconds, the motion-dependent, propagated integration errors projected on the seafloor are similarly periodic
As discussed in Section 2.2.3, the lower limit on suitable window length is just greater than two wave periods, such that the projected wobbles are adequately excluded from the quadratic surface. Implementing this limit on the window length ensures the underlying true, long wavelength seafloor signature is filtered as strongly as possible, at least along track. The consequence of such a temporally short window, however, is less variation in vessel phase over the domain, and thus less independent information is available for local regression. The result is that ambiguities between the integration errors can be more severe. For example, the likelihood of having combinations of roll and pitch too similar for distinguishing x and y-lever arm errors increases; this is explored in Section 3.2. As a result, the more comfortable window extent of four wave periods is implemented throughout the analysis here.

Along-track undulation is of particular significance to the analysis here, as this sees variation in seafloor suitability relative to a quadratic surface, specifically in the rate of change of its seafloor slope and curvature over the domain. Analyzing various undulation wavelengths proves useful in assessing the method’s ability to operate in non-ideal seafloor environments. In addition, the wobbles, or realized depth errors, evolve primarily along-track, and undulation in this direction poses the potential of correlating with them. Seafloor regions which poorly fit the quadratic surface see the unfiltered seafloor trends producing false depth misclosure estimates. The remnant seafloor trends often correlate somewhat with the integration errors, causing spurious, biased estimates ([27], [28]).

The real motion time series used by the swath simulator is observed to have a fairly broadband spectra of roll, though peak oscillation periods are found around 4, 8, 11 and 15 seconds, among others. Pitch, on the other hand is more centered around four seconds. Considering roll to be generally more significant to analysis, the eight second peak is assigned as the characteristic wave period, and a comfortable window length of 32 seconds is implemented, equivalent to 160 meters along-track for a vessel steaming at 5 m/s. Again, the across-track window length averages around four times the regional water depth.

Utilizing the real motion time series, obtained from a 10m survey launch, and defining seafloor’s along-track undulation to have a maximum slope of approximately three degrees (a typical limit on natural unconsolidated fine sediments’ angle of repose), integration error estimates are presented for a number of synthetic seafloors at a host of depths and along-track wavelengths. This is in order to assess the method’s capability in various seafloor “environments” of interest. The maximum slope is defined analytically through the maximized argument of the sinusoidal surface’s along-track derivative:

$$\frac{dz}{dx} = \text{arg max} \left( \frac{d}{dx} \left( A \sin \frac{2\pi x}{L} \right) \right) = \frac{2\pi A}{L} \cdot 1 \ rad = \frac{2\pi A}{L} \cdot \frac{180}{\pi} \ deg,$$

having an amplitude-to-wavelength aspect ratio of 0.05.

Figure 10 presents two local quadratics fit to a 300m seafloor undulation using a 160m, or 4 wave period, along-track window length. While the window length is expected to suitably ignore the high frequency wobbles, it is a bit too long to effectively portray the undulating seafloor. The result is that integration errors (bottom row) as well as the respective adjusted depth misclosures (top middle) are initially grossly inaccurate, apart from motion scaling. Each other estimate is observed to have more than ten percent relative error.
This initial result (red surface and value in plots), however, is only the first local estimate and, as the methodology may be repeated sequentially, successive estimates may contribute to an asymptotic average. Assuming all data to be suitable for analysis, twenty further successively offset local regressions are acquired over the next 40 seconds with thus widely different instantaneous orientation combinations and quality of quadratic surface fit. By compiling the running average of the 20, the asymptotic estimate (black surface and value in plots) is seen to converge dramatically on the truth.

Figure 10: Accuracy of asymptotic integration errors (boxplots, bottom) and quadratic fit estimated using: 1 versus 20 local domains for the regional asymptotic average (red and black bars in boxplots, respectively). The respective surface fits are similarly colored. Blue circles represent each local estimate. The swath corridor is simulated over gently rolling bathymetry, a sinusoid oscillating along-track ($A = 15m, \lambda = 300m, \theta_{max} = 2.86^\circ$). Boxplot limits are $\pm 50\%$ relative error.

As Figure 10 demonstrates, estimates converge to their true values fairly quickly, particularly the lever arms and SSS bias, over this relatively short regional domain of three hundred meters, equivalently to one minute. The remaining estimates are observed to “walk” a fair amount, though seemingly centered about the true values, in red. This is indicative that analyzing more data may assist convergence.

Implementing more data for analysis is highly feasible, as the analysis can be theoretically carried out on all suitable data acquired for mission, and during transit. While only suitable conditions are presented here, to reduce the computational expense of the method to reasonable magnitudes, a data selection scheme should be developed to seek and opportunistically analyze data most suitable for calibration. This is left for future work. In either case, the question is then raised, how long must the dataset be until the asymptotic estimate converges to the truth? In other words, how long until:
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1. the errors are regionally over-identified by a satisfactory number of non-independent data,

2. the errors in the system, here induced by the combination of component vessel orientations, and the variation in seafloor trends occurring within a domain, cancel out.

Figure 11a presents the asymptotic time series of the locally and regionally smoothed estimates for the above 300m wavelength case, while Figure 11b presents the results acquired over a more comfortable 700 meter along-track undulation, closer to the 640m recommended in Section 2.3. The y-axis limits presented for each time series are the 99% confidence interval of each regional, “asymptotic” mean’s standard error for the 300m undulation, in order to enhance comparison of the two cases’ results. This statistic is calculated assuming each successive average to be an independent draw from the population’s distribution. This is of course a significant simplification, as each successive estimate shares the majority of soundings within the adjacent domains. The figure shows that for an along-track wavelength of 300 meters, the asymptotic mean (green line) of the local estimates (blue line) converges to the truth (black line) within approximately three minutes for each error.

As identified in Figure 10, initial estimates are highly inaccurate, however, the asymptotic averages are observed to converge to around 1% relative accuracies within approximately three minutes. The results associated with the 700 meter along-track undulation are far more favorable, as expected. This is directly a result of consistently better seafloor fits, as the seafloor trend is better removed, resulting in less of the seafloor signature leaking into analysis of the periodic wobbles. Imperfect estimates resulting from non-ideal platform motion nonetheless arise, also causing the estimates to “walk” about the true values. “Tuning” of the estimates is thus recommended, through analyzing additional local domains, or sections of swath corridor. While estimates for the 700m case converge to accuracies similar to the 300m case almost instantly, on closer inspection, stable convergence, to a significantly increased estimate accuracy, occurs similarly around three minutes, indicating the influence the input motion plays in convergence.

Figure 11: Asymptotic time series of smoothed, local parameter estimates (blue line), their instantaneous asymptotic average (green line) and their true values (black line) for the case of soundings acquired from a platform driven with real motion, over a 500m deep synthetic seafloor, having along—track undulation of a) 300m b) 700m.

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The results demonstrate that consistent along-track undulations having wavelength greater than 300m and a maximum slope around three degrees should be suitable for analysis, once datasets of suitable extent, here approximately three minutes, are acquired. As the regional domain grows through successive local estimates, acquired only from the growing corridor, the imperfect local estimates contribute to an increasingly accurate asymptotic average.

In a limiting sense, high frequency rugosity, or seafloor roughness, generally having smaller magnitudes and less systematic spatial distribution may inhibit, but is not expected to derail the asymptotic result, assuming its spectrum approaches that of white noise over the asymptotic regional domain. This subsequently results in the method having a robustness to fleeting local combinations of input which can severely bias local, and even short regional estimates. Such combinations are presented in Section 3.2.

The following table presents the asymptotic average and its 99% confidence interval for the 300m and 700m wavelengths undulations at 500 meters water depth illustrated above, as well as for a planar seafloor. These are repeated for regional depths of 50 and 5000 meters, with every dataset acquired using the same, real motion time series input. True error magnitudes are indicated below their respective column headers. Window lengths are four wave periods in each case, equating to the previously discussed 160m. A minimum domain length of two pings is defined in deep water, as the shot receive cycle (around 16 seconds in 5km water) may exceed time domain defined as functions of short wave periods. Error magnitudes are such that they all propagate with peak amplitude of ±0.25% water depth when driven by three degree, eight second sinusoids, as determined through simulation (Fig. 4). Lever arms errors, which are translational and do not scale with depth, are necessarily increased by an order of magnitude in correspondence with each order of magnitude increase in depth, to maintain this relative error magnitude.
Table 1: Simulated results of simultaneously present integration errors when driven by time series of real motion illustrated in Figure 10 and acquired over the described seafloor models. Results are typically more accurate in shallow water conditions where the samples over a given window length, here four wave periods, are increased.

<table>
<thead>
<tr>
<th>Z (m)</th>
<th>λ (m)</th>
<th>ΔL_x</th>
<th>ΔL_y</th>
<th>Δt</th>
<th>Δρ</th>
<th>Δκ</th>
<th>ΔSSS</th>
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<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
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<td>σ_μ</td>
<td>σ_μ</td>
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<td>0.014</td>
<td>-1.000</td>
<td>0.011</td>
<td>-0.020</td>
<td>0.000</td>
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<tr>
<td>300</td>
<td>-0.921</td>
<td>0.583</td>
<td>-0.932</td>
<td>0.640</td>
<td>-0.020</td>
<td>0.001</td>
<td>0.015</td>
</tr>
<tr>
<td>700</td>
<td>-1.004</td>
<td>0.179</td>
<td>-0.998</td>
<td>0.153</td>
<td>-0.020</td>
<td>0.001</td>
<td>0.229</td>
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<tr>
<td>500</td>
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<td>0.314</td>
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<td>0.032</td>
<td>-0.020</td>
<td>0.000</td>
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<tr>
<td>300</td>
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<td>3.443</td>
<td>-10.759</td>
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<tr>
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<tr>
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<td>-0.021</td>
<td>0.003</td>
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</tr>
</tbody>
</table>
Typically, as a result of simply having more information in the same temporal and spatial extents, the shallow water accuracies acquired under identical conditions of vessel motion are usually best. A notable exception is the case of SSS error, however, where the nonlinearity in the error’s across-track depth misclosures do not grow to significant levels, resulting in ambiguity with the roll bias which results from motion scaling, and the estimates are not satisfactorily decoupled.

As is the general case in regression, the more useful, independent information considered in the domain, the more accurate the results. In deep water conditions, reduced point density is a severely limiting factor for the number of soundings available in regression when combined with limited window lengths. This may be adequately balanced, however, by the independence of the soundings as the motion evolves significantly over the extended ping periods. Further, abyssal plains are common features in the deep ocean, where undulations only over enormous wavelengths, on the scale of tens of kilometers depths, are expected and thus window extent may be significantly increased over such bathymetry.

Notably, while high spatial frequency sand waves or rock ridges may have a signature with periodicity similar to some of the wobbles, they are not usually expected to align with the ship track, and therefore wobble, for significant spatial extents. As a result, the asymptotic estimate provided here should, in a limiting sense, be robust to such high frequency features periodically amongst the generally low frequency, sedimented seafloors found in nature.

3.2 – Suitable Vessel Motion

Wave period is not varied between the expected spectrum of 1 and 25 seconds for this section of the analysis, as increased rates only act to increase the magnitude of latency errors and while the vessel goes through more phases of motion in a given time period, the rate at which those phases they are sampled does not necessarily increase. Rather, it is the relationships among the various component orientation magnitudes through time that are of interest for deriving an accurate solution. If component frequencies are significantly different, any domain extended somewhat in time, as done here, will see decoupled amplitudes, and thus are expected to be suitable. Failure of the method due to vessel motion is only likely to arise when the wave induced platform motion has similar spectral characteristics for roll and pitch, the primary drivers behind each integration error’s wobbles.

In order to analyze the suitability of various combinations of vessel motion, component orientations are simulated here using sinusoids oscillating with deliberately slight offsets in period. This results in the component motions continuously “walking by” each other in phase, yet never being exactly identical. This motion is then induced over a seafloor with a 500 meter along-track wavelength. This produces a datasets containing a spectrum of motion and seafloor suitability (Fig. 13), which are the primary drivers behind the method’s success. By monitoring the local estimate variations as a function of undulation phase, it is apparent that the quadratic fit is improved at the crests and troughs (where only one inflexion is present), and is worst on the sloped regions where the rate of change of curvature is highest. As long as the misfit estimates are randomly, or more generally symmetrically distributed, the asymptotic average will converge on the more stable solutions.
Periods of extreme uncertainty arise throughout the “monitoring” of the local parameters estimates, with shocks (Fig. 12, lassoed regions) induced by near multicollinearity of the x-lever and y-lever arm errors, and are exacerbated by poor seafloor fit. These periods of motion multicollinearity are typically more spurious, as they cause a far more severe ambiguity in the source of depth misclosures than the wandering of the local estimates linked to quadratic seafloor misfits. This fleeting multicollinearity induced by roll and pitch being in and out of phase imposes significant “spikes” on the time series of local lever arm estimates, though does extend to the sources of angular errors (errors 3 to 6). In contrast, the angular errors are observed to be more impacted by quadratic surface misfit. Because both the motion multicollinearity and seafloor quadratic misfits are symmetrically distributed over longer time scales, the asymptotic mean is fairly stable through time. This is despite significant variance in the local estimates.

The compelling conclusion is that any combination of motion over the regressed swath corridor, with length a few tens of seconds, is suitable for solution. This is once the impact of motion multicollinearity and seafloor quadratic misfits are asymptotically mitigated as the regional domain length is increased.

IV – Conclusion

A least squares approach has been developed to simultaneously solve for six unknown integration errors in a multi-sector multibeam. The approach has been tested using a simulator that reproduces the sounding output of a three-sector, yaw-pitch-roll stabilized multibeam sonar that operates in typical open-ocean wave conditions. The simulation is run over undulating seafloor terrains with depths ranging from 50m to 5000m, thereby simulating both natural long-
wavelength seafloor curvature and the wide range of ping periods relative to the ocean wave periods.

The method works by first estimating the true long wavelength curvature of the seafloor. This is derived over an along-track length scale that is large compared to the imprint of the motion residuals, but short with respect to changes in seafloor curvature. Thus, prior to estimating the integration errors, the true seafloor shape has to also be estimated. This is achieved by fitting a second order quadratic to the window of acquired soundings. The success of this first estimation relies on their being no actual seafloor roughness with curvature tighter than the length scale implied by the logged window.

Once the seafloor fit is estimated, the 1000 to 100,000 sounding solutions are compared to the seafloor model and the vertical component of their residuals used as input to the least squares estimator.

The estimation is obtained repeatedly as the seafloor model estimate is updated along track. Instantaneous estimates appear stable within approximately 10% of their true value almost immediately. Finer accuracy however, depends on the fit of the seafloor and the magnitude and independence of the driving motions: heave, roll, pitch and yaw. Thus there are periods when the seafloor curvature is poorly estimated and when there is significant correlation between the driving components.

To avoid these limitation an asymptotic average is derived, that represents a running compilation of all instantaneous solutions. In this manner the solution converges on the true integration errors within better than 1% for most cases. The resulting seafloor wobble is correspondingly reduced from typical values of ± 0.25% of depth to < ± 0.01% of depth.

Future intentions for testing this methodology are to apply it to real multibeam data to assess the efficacy of the approach under operational conditions.

References


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